Int. J. Heat Mass Transfer. Vol. 17, pp. 205–214. Pergamon Press 1974. Printed in Great Britain

THERMAL CONTACT CONDUCTANCE; THEORETICAL CONSIDERATIONS

B. B. MIKIĆ
Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. 02139, U.S.A.

(*Received* 4 *August* 1972 *and in revised form* 24 *July* 1973)

Abstract—Thermal contact conductance of nominally flat surfaces in contact was considered. The emphasis of the work is on effect of the mode of deformation on the value of conductance. Explicit expressions for thermal conductance were derived for cases of: (1) Pure plastic deformation (2) plastic deformation of the asperities and elastic deformation of the substrate and (3) pure elastic deformation. The last two important cases are considered for the first time here. Criteria which determine mode of deformation is also presented.

NOMENCLATURE

a, radius of contact;
A_c, actual contact area;
A_a, apparent contact area;
b, radius of an adiabiatic channel for a given contact point;
ΔC, displacement;
E, modulus of elasticity;
E', $(E_1E_2)/[E_2(1-\nu_1^2)+E_1(1-\nu_2^2)]$;
h, thermal conductance;
$\bar{h}$, $\dfrac{n\sigma}{k\tan\theta}$;
H, microhardness;
k, thermal conductivity;
n, number of contacts per unit area;
$\bar{n}$, $n\sigma/\tan^2\theta$;
$\bar{\bar{n}}$, dimensionless shape of a variation with the distance between surfaces in contact;
P, pressure;
P_p, H/P;
P_e, $\dfrac{(\sqrt{2})P}{E'\tan\theta}$;
Q, heat rate;
$Q(\eta)$, $1/2\ \mathrm{erfc}\,(\eta/\sqrt{2})$;
$\tan\theta$, the mean of absolute slope of a profile;
Y, distance between mean planes of two surfaces in contact;
$Z(\eta)$, $\dfrac{1}{\sqrt{2\pi}}\exp(-\eta^2/2)$;
α, $\dfrac{\sigma}{\tan\theta}\dfrac{\Sigma a_i}{Aa}$;
ζ, dimensionless distance between mean planes when an asperity first came in contact;
ν, Poisson's ratio;
η, Y/σ;
ρ, radius of curvature of an asperity;
$\bar{\rho}$, $\dfrac{\rho\tan^2\theta}{\sigma}$;
σ, standard deviation of profile height;
ψ, contact resistant factor, equation (2).

INTRODUCTION

IN THE last thirty years much published experimental and theoretical work has been devoted to problems related to thermal contact conductance. The basic mechanism is understood (e.g. [1–5]). Excellent experimental data have been reported, notably by Fried [6–8]. Several partially successful correlations have been attempted, but a general correlation has not been developed. This is not surprising in view of the large number of parameters and phenomena which could affect the results. The loading hysteresis effect was considered (e.g. in [15, 16]), and was shown to be significant but not always producing the same effect [16]. Plating was treated in [6] and [17]. Several notable publications on directional effects have become available in recent years [18, 19]. Problems associated with rough spheres with planes, and rough and smooth contact in bolted joint geometry are investigated in [20] and [21].

No list of significant contributions in the area of contact resistance can be complete without works in the area of surface description and surface behavior in contact. The most notable contributions, which represent only a small fraction of the available literature in the area, are [22–28]. In fact, it should be recognized that most difficulties in understanding and explaining certain observed phenomena can be related to deformation. The main questions which are not satisfactorily answered are related to mode of asperities deformation and development of prediction for thermal resistance when the asperities deformation is elastic.

This work represents an attempt to bring some facts and evidence which could help in answering these questions. In particular, considering nominally flat surfaces in contact, individual simple expressions for thermal contact conductance with assumed plastic and elastic deformation will be given. From the basic model, it will be concluded what is the probability that each will occur. Furthermore, a re-examination of the plastic model, taking into account the effect of plastic flow in the region of contact area on the surface displacement outside the contact, will be made. In addition, the effect of the elastic deformation of the substrate below the plastic deformation of the asperity on the surface parameters will be considered. The magnitude and region of significance of these effects will be indicated and compared with the available experimental evidence in subsequent work [29].

Basic relations

This work is restricted to the case of rough nominally flat surfaces in contact in a vacuum under conditions of negligible radiation. The extension of the results to the cases not covered by the above model is reasonably straightforward [30, 31]. The heat flow under steady state conditions across the joint will be distributed among the contact spots of different sizes existing at the contact plane. With the assumption that the contact point is circular with a radius a_i, one can write for each individual contact:

$$Q_i = 2ka_i\,\Delta T_c \frac{1}{\psi_i}. \tag{1}$$

ΔT_c is the temperature drop across the interface (obtained by extrapolating the respective temperature profiles on the two sides of the interface). For *geometrically* similar contact, i.e. when the contact plane is the surface of symmetry, ΔT_c is the same for all contact points.

ψ_i is a geometrical factor equal to unity for a single contact belonging to an infinite apparent area. The value of the geometrical factor has been considered by several authors, analytically in [1–5], and using a numerical solution in [2]. For "appropriately" distributed contacts, i.e. when the contact point is in the center of the adiabatic cylinder, in [4] a simple expression, which closely approximates the analytical and numerical solution, is given as

$$\psi_i = \left(1 - \frac{a_i}{b_i}\right)^{1\cdot 5} \tag{2}$$

where b_i is the radius of the adiabatic cylinder.

Heat flow per unit apparent area (A_a) easily follows from (1) as

$$\frac{Q}{A_a} = \frac{\Sigma Q_i}{A_a} = 2k\,\Delta T_c \sum \frac{a_i/A_a}{\psi_i}$$

or, neglecting variations in ψ_i from contact to contact compared with variation in a_i,

$$\frac{Q}{A_a} \simeq \frac{2k\,\Delta T_c}{\psi} \Sigma a_i/A_a \tag{3}$$

where

$$\psi = [1 - (a_i/b_i)_{\text{av}}]^{1\cdot 5} = (1 - \sqrt{A})^{1\cdot 5}$$

where A is the *fraction* of actual area in contact.

The thermal contact conductance then follows from equation (3):

$$h \equiv \frac{Q/A_a}{\Delta T_c} = \frac{2k}{\psi} \Sigma\, a_i/A_a. \tag{4}$$

The value of $\Sigma a_i/A_a$ can be obtained from consideration of asperities deformation.

The surface parameters which appear in the expression for $\Sigma a_i/A_a$ depend on the way one describes the surface. Presently there is no common approach for describing surfaces. Three most frequently used approaches describe a rough surface by:

(i) distribution of peak height and asperity shape. All asperities are assumed to be of identical shape (e.g. [22–24]); or, two or more generations of asperities of similar shape but different size scale are superimposed onto each other [26].
(ii) distribution of surface height (i.e. height of all points on a surface and not just peaks) and autocorrelation function (e.g. [27]).
(iii) distribution of surface height and distribution of surface slope (e.g. [4, 15]).

Usually in all models, the distribution of surface height or peaks (depending on the description) is assumed to be Gaussian. Either is acceptable according to the extensive experimental observation of actual surfaces [28]. However, the second part of surface description is not consistent in the three approaches. The first approach assumption of identical shape of asperities is inconsistent with the other two descriptions which imply that the expected peak curvature is a function of the peak height. Here the third approach, which is one of the two less restrictive descriptions, will be adopted, i.e. the surfaces are described here by standard deviation (σ) for combined height distribution and the average of the absolute value of the slope ($\tan\theta$) for the combined slope distribution.

Using these two parameters, one can rewrite equation (4) in dimensionless form as

$$\bar{h} = \frac{2\alpha}{\psi} \tag{5}$$

where

$$\bar{h} \equiv \frac{h\sigma}{k \tan\theta}$$

and

$$\alpha \equiv \frac{\sigma}{\tan\theta} \sum a_i/A_a.$$

Relation (5) is derived without any reference to mode of deformation; the evaluation of parameters ψ and α will, however, depend on a particular mode of deformation.

CONTACT PARAMETERS

Plastic deformation

(a) *Geometrical model.* In the load range of practical interest, it is usually sufficient to treat plastic deformation of rough surfaces in contact as a pure geometrical interaction between the asperities of contacting surfaces. With the concept of only one surface rough, having an "equivalent roughness" in contact with a smooth one, the above model of geometrical interaction implies that the asperities are flattened (or equivalently penetrated into the smooth surface) without any change in shape of the part of both surfaces not yet in contact. Therefore bringing the two surfaces (Fig. 1) together within a distance Y is equivalent to slicing off the top of the asperities at a height Y above the mean plane.

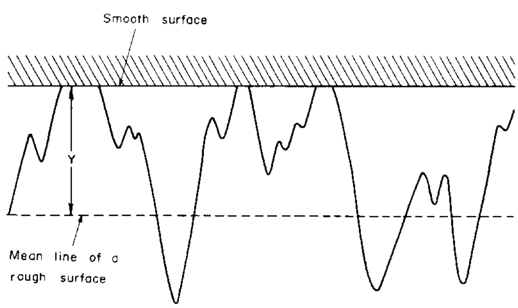

FIG. 1. Pure plastic contact.

In [4], using a model such as this, and describing the surfaces with Gaussian distribution of the height, and random, but independent of the height, distribution of the slope, important contact parameters were expressed as a function of separation distance.

The fraction of actual area in contact is given as:

$$A = \tfrac{1}{2}\operatorname{erfc}(\eta/\sqrt{2}) \equiv Q(\eta) \qquad \eta \equiv \frac{Y}{\sigma} \tag{6}$$

and the sum of the contact radii as

$$\alpha = \frac{1}{4\sqrt{(2\pi)}} \exp\left(-\frac{\eta^2}{2}\right) \equiv \frac{Z(\eta)}{4}. \tag{7}$$

Both $Z(\eta)$ and $Q(\eta)$ are tabulated in standard mathematical tables.

Furthermore, the actual contact pressure in this mode of deformation is assumed to be equal to the microhardness of the softer material in contact, i.e.

$$A = \frac{P}{H} \equiv P_p. \tag{8}$$

From equations (6) and (8) it follows that $\eta = Q^{-1}(P_p)$ and from equation (7) that $\alpha = \frac{1}{4}Z[Q^{-1}(P_p)]$ which together with equations (5) and (3) yield a closed form expression for dimensionless thermal contact resistance:

$$\bar{h}_p = \frac{1}{2}\frac{Z[Q^{-1}(P_p)]}{(1-\sqrt{P_p})^{1\cdot 5}}. \tag{9}$$

Equation (9) gives $\bar{h}_p$ as a function of dimensionless pressure only. In Fig. 2, equation (9) is plotted (line labeled with $\gamma = 0$).

The following simple equation approximates relation (9) remarkably well:

$$\bar{h}_p = 1{\cdot}13\, P_p^{0\cdot 94} \text{ or } h_p = 1{\cdot}13 \frac{k\tan\theta}{\sigma}(P/H)^{0\cdot 94}. \tag{10}$$

The above form is only slightly different from the expression previously suggested in [4].

(b) *Plastic flow of material considered.* Pure geometrical interaction obviously cannot be a proper model for very large pressure (approaching hardness) or, equivalently, very low separation approaching $\eta \to 0$. This is evident, for example, from equation (6), which states that when the mean distance between the contacting planes approaches zero the fraction of area in contact approaches only one-half. This is a direct consequence of neglecting the flow of displaced material. Pullen and Williamson [32] experimentally investigated plastic flow under large loads. They concluded that the volume of material remained constant and that the material plastically displaced appears as a uniform rise over the entire surface. Since the *uniform rise* will not affect the *shape* fo the surface outside the contact area, equations (6) and

(7) still can be used by replacing η with an imaginary separation λ which is related to the real separation η through a geometrical consideration based on the model of uniform rise. In [32] such an analysis was performed, and the following relations were obtained:

$$\left.\begin{aligned}\eta &= \lambda - \lambda\, Q(\lambda) - Z(\lambda) \qquad \lambda > 0\\ \eta &= Z(\lambda) - |\lambda|\, Q(\lambda) \qquad \lambda < 0.\end{aligned}\right\} \tag{11}$$

For large positive values of λ ($\lambda > 1$), $\lambda \simeq \eta$; for $\lambda \to -\infty, \eta = 0$, and hence from relation (6) (recalling that with this modification one should replace η with λ), when separation is zero ($\lambda = -\infty$), the fraction of area in contact is unity.

In the same investigation, Pullen and Williamson [32] found that the contact area due to the interaction of microcontacts is not proportional to the normal load; they proposed as a good approximation:

$$A = \frac{P_p}{1 + P_p}. \tag{12}$$

It can be seen that modification due to the plastic flow in expressions for thermal conductance (9) and (10) is only in replacement of P_p with A, or using (12) with $[P_p/(1 + P_p)]$, i.e. for equation (10)

$$h_p = 1{\cdot}13\,\frac{k \tan\theta}{\sigma}\left(\frac{P}{H + P}\right)^{0{\cdot}94}. \tag{10a}$$

This modification will tend to reduce the value of h_p at relatively high loads, whereas the effect is negligible for normal loads.

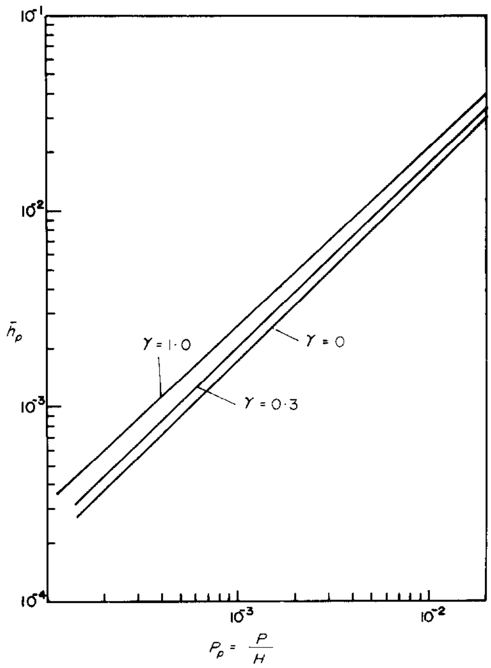

Fig. 2. Thermal contact conductance vs interface pressure for plastic deformation.

(c) *Plastic deformation with correction for elastic displacement.* Both plastic deformation models described above do not consider the effects of elastic deformation under the contact points. These effects would be negligible either if the modulus of elasticity of the surface were very large or if the distance between neighboring contact points were so small that the elastic displacement is approximately the same for any part of the surface in contact. Actually these never would be the case, and elastic displacement beneath a contact point would always be larger than for the area outside the contact. As a consequence of this, the area of contact for an individual asperity would be smaller at a given separation than for the pure plastic deformation model. This of course would be true for all contact points. Therefore, for the whole surface at a given separation, the contact area would be smaller, or inversely, to achieve the same area in contact, the separation between the contacting surfaces must be smaller.

By assuming that the deformation of individual asperities does not affect the displacement of neighboring asperities, i.e. that each asperity deformation can be considered independently, one can conclude that the number of contact points at a given separation would remain unchanged. Consequently, it follows from the above that the same area in contact, with the proposed modification, would be formed with larger number of contacts. This of course will affect the value of the thermal contact conductance.

In the appendix, deformation aspects of the above phenomena are considered. Figure 3 gives the results of the analysis: the values of A/A_0 and α/α_0 as a function of the parameter $\gamma \equiv H/(E' \tan\theta)$ are plotted (the subscript "0" refers to pure plastic model values); and

$$\frac{1}{E'} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}.$$

It can be seen that the area in contact is more affected by the introduced modification than is the parameter α. Furthermore, the modification in α for a given γ is insensitive to separation. The effect of the separation distance η on A, for a given γ, is relatively larger, but absolutely is still very weak. The two bounds shown

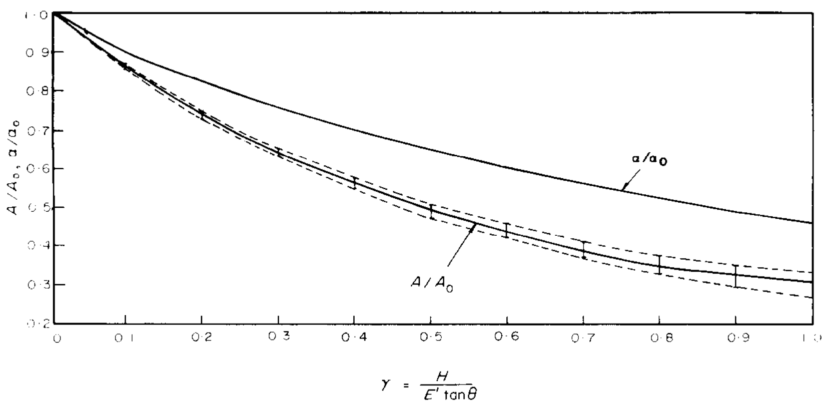

FIG. 3. Effect of elastic deformation of substrate on contact parameters.

above and below the line represent $\eta = 1$ and $\eta = 3$, respectively. They cover the pressure range of practical interest; the upper bound represents $\eta = 1$.

One can then approximate A and α for the modified model which includes elastic deformation of the substrate as

$$A = \phi_1(\gamma)\, Q(\eta)$$

and

$$\alpha = \phi_2(\gamma) \frac{Z(\eta)}{4} \tag{13}$$

where ϕ_1 and ϕ_2 could be read from Fig. 3. The two lines could be approximated by the following analytical expressions:

$$\begin{aligned} \phi_1(\gamma) &= 1/(1 + 2{\cdot}5\, \gamma^{1{\cdot}2}) \\ \phi_2(\gamma) &= 1/(1 + 1{\cdot}05\gamma). \end{aligned} \tag{14}$$

From equations (5)–(7), (13) and (14) follows the relation for $\bar{h}_p$ with the inclusion of the elastic displacement modifications as

$$\bar{h}_p = \frac{Z(Q^{-1}[(1 + 2{\cdot}5\gamma^{1{\cdot}2})P_p])}{2(1 + 1{\cdot}05\gamma)(1 - \sqrt{P_p})^{1{\cdot}5}}. \tag{15}$$

In Fig. 2, $\bar{h}_p$ is plotted for values of $\gamma = 0$, 0·3 and 1 respectively.

It can be seen from the figure that increase in γ, i.e. increase of elastic deformation, increases $\bar{h}_p$. This effect is more pronounced at lower loads. In the vicinity of P_p value of 10^{-3}, one can estimate this increase from the following relation:

$$\bar{h}_p = (\bar{h}_p)_{\gamma=0}(1 + 0{\cdot}6\gamma). \tag{16}$$

In order to determine the effect of the considered correction, one must know the value of γ. For example, using the properties of stainless steel and $\tan\theta = 0{\cdot}1$ (an average value for bead blasted surfaces), the calculated value for γ is 0·245, which gives increase in $\bar{h}_p$ at lower pressure range of about 17 per cent. For lower values of $\tan\theta$, this increase would be larger. One should point out that at the high pressures, the actual effect of the elastic deformation of the substrate should be even less than indicated in the figure. This is because the modeling considered displacement of individual asperities independently. At high pressures when the average distance between the contacts is not too large, the displacements cannot be considered independently, i.e. the displacement of one contact will also affect the neighboring asperities. This will cause more uniform elastic displacement of the asperities and decrease the effect of elastic deformation of the substrate on thermal contact conductance (if all asperities are equally displaced, the effect would be zero).

Elastic deformation

For an asperity in contact with a rigid flat surface in elastic deformation, the contact area can be related to the displacement using the Hertzian theory [33] as

$$(\pi a_i^2)_e = \pi \Delta C_i \rho_i. \tag{17}$$

If the deformation were purely plastic, one gets from simple geometrical considerations

$$(\pi a_i^2)_p = 2\pi \Delta C_i \rho_i. \tag{18}$$

Comparison between equations (17) and (18) shows that at the same separation η between the contacting surfaces, the contact area in purely plastic deformation for any specific asperity would be twice the contact area in elastic deformation. Since this is true for all asperities, the same holds true for the entire contact area. Hence, at the same η,

$$\frac{A_{\text{elastic}}}{A_{\text{plastic}}} = \frac{1}{2}$$

or from equation (6), for the elastic deformation,

$$A = \tfrac{1}{4}\,\text{erfc}\,(\eta/\sqrt{2}) = \tfrac{1}{2}Q(\eta). \tag{19}$$

Similarly, from equation (7), one gets for the sum of all contact radii:

$$\alpha = \frac{1}{8\sqrt{\pi}}\exp\left(-\frac{\eta^2}{2}\right) = \frac{1}{4\sqrt{2}}Z(\eta). \tag{20}$$

A number of investigators (e.g. [22, 24]) have reported that even in the elastic mode of deformation the actual area in contact is approximately proportional to the load; the more complicated the model used for description of the rough surface, the closer direct proportionality was obtained. Using the description adopted in this work, the direct proportionality was actually exactly obtained [34]; specifically,

$$A = \frac{P\sqrt{2}}{E'\tan\theta} \equiv P_e \tag{21}$$

where P_e could be related to P_p and γ as $P_e = \sqrt{2}P_p\gamma$.

From equations (5) and (19)–(21) follows an explicit relation for $\bar{h}$ in elastic deformation (denoted further with $\bar{h}_e$) as

$$\bar{h}_e = \frac{1}{2\sqrt{2}}\frac{Z[Q^{-1}(2P_e)]}{(1-\sqrt{P_e})^{1\cdot 5}}. \tag{22}$$

In Fig. 4, $\bar{h}_e$ is plotted as a function of P_e. For comparison on the same figure $\bar{h}_p$ for purely plastic deformation is shown as a function of P_p. The following simple expression approximates the plot accurately:

$$\bar{h}_e = 1{\cdot}55 P_e^{\,0\cdot 94} \tag{23}$$

or

$$h_e = 1{\cdot}55\frac{k\tan\theta}{\sigma}\left(\frac{P\sqrt{2}}{E'\tan\theta}\right)^{0\cdot 94}. \tag{23a}$$

It can be seen from equation (23a) that h_e is a very weak function of $\tan\theta$ ($\sim\tan\theta^{0\cdot 06}$). If we substitute $\tan\theta = 0{\cdot}1$ (the average value for blasted surfaces), equation (23a) simplifies to

$$h_e = 1{\cdot}9\frac{k}{\sigma}\left(\frac{P}{E'}\right)^{0\cdot 94}. \tag{24}$$

Relatively large deviations in $\tan\theta$ from the assumed value of 0·1 would not appreciably change the above relation.

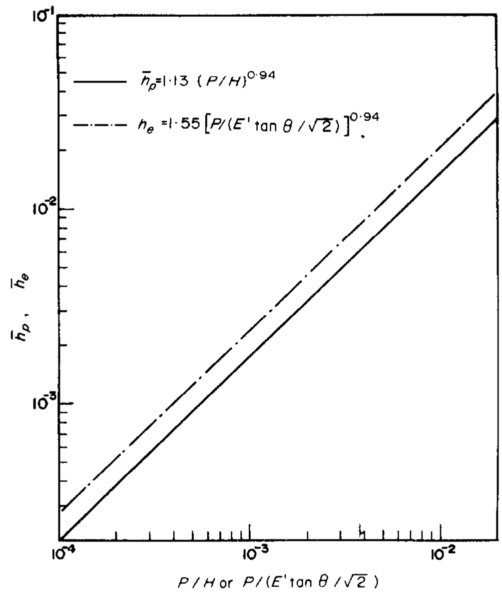

FIG. 4. Thermal contact conductance vs interface pressure for elastic ($\bar{h}_e$) and plastic ($\bar{h}_p$) deformation of asperities.

There are a few obvious conclusions which can be drawn from the results of this analysis.

(i) The predicted slope of the thermal contact conductance vs pressure is the same for purely plastic and elastic deformation. This slope is slightly less than unity.

(ii) If $P_e = P_p$, the thermal contact conductance is higher for the elastic deformation. However, one should be very careful not to draw any further direct conclusions from that. When $P_e = P_p$, it does not mean that the interface pressures are the same; we recall that

$$P_e \equiv \frac{(\sqrt{2})P}{E'\tan\theta} \text{ and } P_p \equiv \frac{P}{H}.$$

Hence $P_e = P_p$ would imply the same interface pressure only if

$$H = \frac{E'\tan\theta}{\sqrt{2}},\ \text{i.e. } \gamma \equiv \frac{H}{E'\tan\theta} = \frac{1}{\sqrt{2}}.$$

Looking differently at the plot, one can ask at what P_e/P_p, at the same interface pressure, the two approaches will give the same h.

From equations (23) and (10), one gets for this case $P_e/P_p = 0{\cdot}734$ or $\gamma = 0{\cdot}52$. Furthermore, for $\gamma < 0{\cdot}52$, at the same pressure, the purely plastic deformation model will yield a higher h. For $\gamma > 0{\cdot}52$, the opposite would be true.

The above observations are only formal interpretation of the results shown in Fig. 4, and they do not provide any explicit indication of what kind of deformation would actually take place for two specific interfaces in contact. This is considered in the next section.

Probability for plastic flow to occur

A priori assumption of mode of deformation obviously could lead to wrong conclusions. For example, if one had assumed elastic deformation of asperities in contact for surfaces with $\gamma \leqslant 0{\cdot}52$, one would have concluded, as shown in the previous section, that h would be lower than if plastic deformation were assumed. This seems surprising, since one would intuitively expect that elastic deformation, as a consequence of higher area in contact, would yield higher h.

The higher area in contact, however, would occur only if contact pressure is lower than hardness (which is contact pressure in plastic flow). On the other hand, elastic deformation cannot exist if the contact pressure is larger than the hardness. Actually when contact pressure exceeds $1{\cdot}1\ Y_0$ [35], where Y_0 is the yield strength in pure tension ($Y_0 \simeq H/3$), the elastic limit will just be exceeded. Returning to our example, one can calculate the contact pressure for $\gamma = 0{\cdot}52$. From equation (21), the average contact pressure in elastic deformation is

$$P_c = \frac{E' \tan\theta}{\sqrt{2}}$$

or, rewriting,

$$P_c = \frac{E' \tan\theta}{H\sqrt{2}} H = \frac{H}{\gamma\sqrt{2}} = 1{\cdot}36H.$$

Hence, for $\gamma = 0{\cdot}54$, elastic deformation would give contact pressure larger than hardness. This, of course, implies that for $\gamma \leqslant 0{\cdot}54$, one cannot have elastic deformation. Actually, all contacts will not have the same contact pressure, although the average contact pressure would remain constant. One can carry an analysis to determine contact pressure distribution over the whole contact area. In [34] such an analysis was performed. Figure 5 gives the result, where the fraction of area in contact with contact pressure larger than P_c was plotted as a function of dimensionless P_c. The result is not very sensitive to the pressure level. ($\eta = 1$ and 2 respectively represent the results for the different separations, i.e. two different pressure levels.)

From the figure, one can see that when $P_c = 1{\cdot}1\,Y_0$,

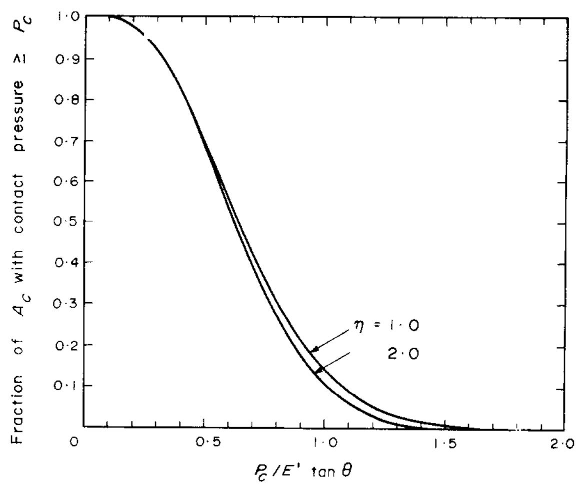

Fig. 5. Contact pressure distribution.

i.e. for

$$\frac{1{\cdot}1\,Y_0}{E' \tan\theta} \geqslant 1{\cdot}1$$

90 per cent of the actual area will have contact pressure less than $1{\cdot}1\,Y_0$, or substituting $Y_0 = H/3$, one can conclude that when $\gamma \geqslant 3$ the deformation will be predominantly elastic. If one assumes that the elastic deformation can proceed for contact pressure even larger than $1{\cdot}1\,Y_0$, one would get an estimate from the figure that approximately 90 per cent or more of the contact area would have contact pressure larger than hardness and hence would be in the plastic mode of deformation when $\gamma \leqslant 0{\cdot}33$.

In conclusion, the mode of deformation depends on material properties (H and E') and the shape of the asperities ($\tan\theta$); it is not sensitive to the pressure level.

DISCUSSION AND CONCLUSIONS

For rough nominally flat surfaces in contact thermal contact resistance was explicitly evaluated for assumed pure plastic deformation, plastic deformation of asperities and elastic deformation of the substrates and pure elastic deformation. In addition, a criterion determining mode of deformation for given surfaces in contact (given geometry, and materials) was given. The surfaces were described with Gaussian distribution of height and random distribution of the slope.

The case of plastic deformation of the asperities with elastic deformation of the substrate and pure elastic deformation case were solved explicitly for thermal contact resistance for the first time here.

The slope of thermal contact resistance vs load is the asperities, whereas in pure elastic deformation The surface parameters which affect the value of thermal contact resistance in plastic flow are r.m.s. of the surface roughness (σ) and the average slope of the asperities, whereas in pure elastic deformation only σ plays an important role. The mode of deformation, however, is determined by the value of parameter $\gamma = H/(E' \tan \theta)$. Deformation would be predominantly elastic for $\gamma \geqslant 3$ and predominantly plastic for $\gamma \leqslant 1/3$. The inclusion of elastic deformation of the substrate in plastic deformation model causes the increase in value of thermal conductance. This effect is stronger at lower load and hence causes change in conductance vs load slope yielding a lower slope at the lower pressure level.

REFERENCES

1. T. N. Cetinkale and M. Fishenden, Thermal conductance of metal surfaces in contact, Int. Conf. of Heat Transfer, Inst. Mech. E., pp. 271–275, London (1951).
2. A. M. Clausing, Some influences of macroscopic constrictions on the thermal contact resistance, NASA Rep. No. ME-TN-242-2, Univ. of Illinois (1965).
3. L. C. Roess, Theory of spreading conductance, Appendix A of an unpublished report of the Beacon Laboratories of Texas Company, Beacon, New York.
4. M. G. Cooper, B. B. Mikic and M. M. Yovanovich, Thermal contact conductance, *Int. J. Heat Mass Transfer* **12**, 279–300 (1969).
5. R. Holm, *Wiss. Veroeff Siemens-Werken* **7**(2), 217–258 (1929).
6. E. Fried, Study of interface thermal contact conductance, General Electric Co. Report 65 SD4395 (1965).
7. E. Fried, Metallic interface thermal conductance, paper presented at NPL Thermal Conductivity Conference, London (1964).
8. E. Fried and F. A. Costello, Interface thermal contact resistance problem in space vehicles, *J. Am. Rocket Soc.* **32**, 237–243 (1962).
9. R. G. Wheeler, Thermal conductance of fuel element materials, USAEC Report HW 60343 (1959).
10. W. J. Graff, Thermal conductance across metal joints, *Machine Design* **32**, 166–172 (1960).
11. A. C. Rapier, T. M. Jones and J. E. McIntosh, The thermal conductance of uranium dioxide/stainless steel interfaces, *Int. J. Heat Mass Transfer* **6**, 397–416 (1963).
12. T. N. Veziroglu, Correlation of thermal contact conductance experimental results, U. of Miami Mechanical Engineering Dept. Report on NASA Grant NGR 10-007-010- Sub 11 (1967).
13. C. L. Tien, A correlation for thermal contact conductance of nominally flat surfaces in a vacuum, *Proceedings 7th Thermal Conductivity Conference*, pp. 755–759. U.S. Bureau of Standards (1968).
14. T. R. Thomas and S. D. Probert, Correlations for thermal contact conductance *in vacuo*, ASME Paper No. 71-HT-AA.
15. B. Mikic, Analytical studies of contact of nominally flat surfaces; effect of previous loading, *J. Lubrication Technol.* 451–456, (October 1971); ASME Paper No. 71-Lub-M.
16. L. Carruth, Jr., Thermal conductance of arbitrary surfaces against a reference surface, Ph. D. Thesis, H. A. Blum, Supervisor, Southern Methodist University (1970).
17. B. B. Mikic and G. Carnasciali, The effect of thermal conductivity of plating material on thermal contact resistance, *J. Heat Transfer* **92** 475–483 (1970).
18. A. M. Clausing, Heat transfer at the interface of dissimilar metals: the influence of thermal strain, *Int. J. Heat Mass Transfer* **9**, 791–801 (1966).
19. D. V. Lewis and H. C. Perkins, Heat transfer at the interface at stainless steel and aluminum: the influence of surface conditions on the directional effect, *Int. J. Heat Mass Transfer* **11**, 1371–1383 (1968).
20. R. T. Roca and B. B. Mikic, Thermal contact resistance in a non-ideal joint, Heat Transfer Laboratory Report No. DSR 71821-77 on NASA Contract No. NAS 8-24867, Engineering Projects Laboratory, Mechanical Engineering Department, MIT, Cambridge, Mass. 02139.
21. R. T. Roca and B. B. Mikic, Thermal conductance in a bolted joint, AIAA Paper No. 72-282, presented at AIAA 7th Thermophysics Conference, San Antonio, Texas, 10–12 April 1972.
22. J. A. Greenwood and J. B. P. Williamson, Contact of nominally flat surfaces, *Proc. R. Soc., Lond.* **295**, 300–319 (1966).
23. J. A. Greenwood, The area of contact between rough surfaces and flats, *J. Lubrication Technol.* **89F** (1), 81–91 (1967).
24. J. A. Greenwood and J. H. Tripp, The contact of two nominally flat surfaces, *Proc. Inst. Mech. Engng* **185** (48/71), 625–633 (1971).
25. J. F. Archard, Elastic deformation and the laws of friction, *Proc. R. Soc., Lond.* **243A**, 190 (1957).
26. J. F. Archard, Single contacts and multiple encounters, *J. Appl. Phys.* **32**, 1420 (1961).
27. D. J. Whitehouse and J. F. Archard, The properties of random surfaces in contact, Paper presented at ASME Winter Annual Meeting, Los Angeles, November 1969, Surface Mechanic, ASME Publication, New York, pp. 36–37 (1969).
28. J. B. P. Williamson, J. Pullen and R. T. Hunt, The shape of solid surfaces, Paper presented at ASME Winter Annual Meeting, Los Angeles, November 1969, Surface Mechanic, ASME Publication, New York, pp. 24–35 (1969).
29. B. Mikic, Thermal contact resistance: experimental results, in preparation.
30. M. M. Yovanovich, Overall constriction resistance between contacting rough wavy surfaces, *Int. J. Heat Mass Transfer* **17**, 1621–1632 (1966).
31. B. B. Mikic, Thermal constriction resistance due to non-uniform surface conditions; contact resistance at non-uniform interface pressure, *Int. J. Heat Mass Transfer* **13**, 1497–1500 (1970).
32. J. Pullen and J. B. P. Williamson, *Proc. R. Soc., Lond.* **327A**, 159–173 (1973).
33. S. Timoshenko and J. N. Goodier, *Theory of Elasticity*, 2nd Ed. McGraw-Hill, New York (1951).
34. B. B. Mikic and R. T. Roca, On elastic deformation of rough surfaces in contact, to appear in *J. Lubrication Technol.*
35. F. D. Bowden and D. Tabor, *The Friction and Lubrication of Solids*. Oxford University Press (1950).

APPENDIX

Plastic Deformation with Modification due to the Elastic Deformation of the Substrate

When a load is applied to an asperity which is plastically deformed, the displacement which results is a combination of plastic deformation of the tip and elastic deformation of the asperity substrate. If one assumes that the material has an infinite modulus of elasticity, the elastic contrlbution is zero. For this case (see Fig. A.1), the displacement is

$$\Delta C_p = \frac{a_i^2}{2\rho_i}.$$

The pressure over the contact area, since the main mode of deformation is assumed to be plastic, is H (hardness of the softer material of the two bodies in contact). If at this point we remove the restriction of infinite modulus of elasticity, the displacement would increase by an additional ΔC_e (Fig. A.1).

Calculating a_i from equations (A.2) and (A.3) and expressing it in dimensionless form, one gets

$$\bar{a}_i = \left(\frac{\pi^2}{\gamma}\rho^2\gamma^2 + 2\bar{\rho}(\zeta - \eta)\right)^{\frac{1}{2}} - \frac{\pi}{2}\bar{\rho}\gamma \qquad \text{(A.4)}$$

where

$$\bar{a}_i \equiv \frac{a_i}{\sigma}\tan\theta; \qquad \gamma \equiv \frac{H}{E'\tan\theta}; \qquad \bar{\rho} = (\rho/\sigma)\tan^2\theta.$$

If the number of contact points per unit area which originate between ζ and $\zeta + \Delta\zeta$ is denoted by n, one can express the fraction of area n contact and α at a given separation η as

$$A = \pi\int_{\zeta}^{\infty}\bar{a}_i^2\bar{n}'\,d\zeta \qquad \text{(A.5)}$$

and

$$\alpha = \int_{\zeta}^{\infty}\bar{a}_i\bar{n}'\,d\zeta \qquad \text{(A.6)}$$

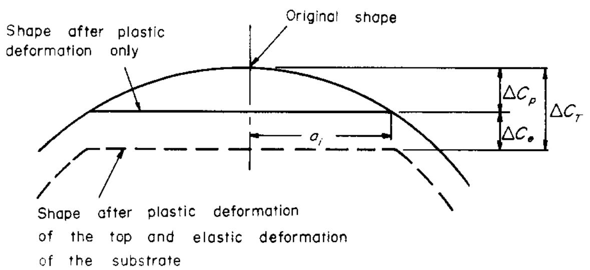

FIG. A1. An asperity in plastic deformation with elastic deformation of the substrate.

The total displacement in this case would be

$$\Delta C = \Delta C_p + \Delta C_e.$$

To calculate ΔC_e one can consider elastic displacement of a disc of radius a_i with pressure over it of magnitude H. The result is given in [33], where

$$\Delta C_e = \frac{\pi}{2}\frac{a_i H}{E'}. \qquad \text{(A.1)}$$

A different approach would be to consider what should be the recovery of the asperity when one removes the load. Since the recovery would be reversible elastic deformation, one can conclude that for the same amount the contact area was elastically displaced during the first loading approach. The recovery was calculated in [15]. Since the recovery would be different for different parts of the contact area (πa_i^2), one can estimate the elastic displacement by averaging the recovery over the contact area. This procedure would yield a result only a few per cent different from the one given by equation (A.1). The former will be used in further analysis.

Therefore:

$$\Delta C = \frac{a_i^2}{2\rho_i} + \frac{\pi}{2}\frac{a_i H}{E'}. \qquad \text{(A.2)}$$

If η is the dimensionless distance between the mean planes and ζ is the dimensionless distance when the asperity first came in contact, then

$$\Delta C = \sigma(\zeta - \eta). \qquad \text{(A.3)}$$

where

$$\bar{n}' \equiv \frac{\bar{n}'\sigma^2}{\tan^2\theta}.$$

Equation (A.5) can be further developed, using (A.1) and (A.2) as

$$A = 2\pi\int_{\eta}^{\infty}(\zeta - \eta)\,\bar{\rho}\bar{n}'\,d\zeta - \pi^2\gamma\int_{\eta}^{\infty}\bar{a}_i\,\bar{\rho}\bar{n}'\,d\zeta. \qquad \text{(A.7)}$$

The value of the first integral is clearly the same as in purely plastic deformation; hence

$$A = Q(n) - \pi^2\gamma\int_{\eta}^{\infty}\bar{a}_i\,\bar{\rho}\bar{n}'\,d\zeta. \qquad \text{(A.8)}$$

The second term on the right hand side is the correction due to the elastic deformation of the substrate. The expressions for $\bar{n}'$ and $\bar{\rho}$ are found in [15] as

$$\left.\begin{aligned}\bar{\rho}\bar{n}' &= \frac{\zeta}{2\pi}Z(\zeta)\\ \text{and}\qquad \bar{\rho} &\simeq \frac{4}{\pi^{\frac{3}{2}}}\frac{1}{\zeta}.\end{aligned}\right\} \qquad \text{(A.9)}$$

A and α, for different values of η ($\eta = 1$ to $\eta = 3$ in increments of 0·5) were calculated on an IBM 1130 computer as a function of γ (from $\gamma = 0$ to $\gamma = 1{\cdot}0$ in increments of 0·1) from equations (A.4), (A.6), (A.8) and (A.9).

Figure 3 shows the results in the form A/A_0 and α/α_0. α/α_0 is not sensitive to values of η, and A/A_0 is only weakly dependent on η.

CONDUCTANCE DE CONTACT THERMIQUE; CONSIDERATIONS THEORIQUES

Résumé—On considère la conductance thermique de surfaces planes en contact. Ce travail concerne particulièrement l'effet du mode de déformation sur la valeur de la conductance. On donne des expressions pour la conductance thermique dans les cas suivants: (1) Déformation plastique pure, (2) Déformation plastique des aspérités et déformation élastique du substrat, (3) Déformation élastique pure. Les deux derniers cas importants sont considérés pour la première fois ici. On présente aussi un critère qui détermine le mode de déformation.

THERMISCHER KONTAKTLEITWERT, THEORETISCHE UNTERSUCHUNGEN

Zusammenfassung— Es wurde der thermische Kontaktleitwert von einander berührenden ebenen Oberflächen untersucht. Die Betonung dieser Arbeit liegt auf der Abhängigkeit der Grösse des Leitwertes von der Art der Verformung. Eindeutige Beziehungen für den thermischen Leitwert wurden für folgende Fälle hergeleitet:

(1) rein plastische Verformung
(2) plastische Verformung der Rauheiten und elastische Verformung des Grundprofils
(3) rein elastische Verformung.

Die zwei letzteren wichtigen Fälle wurden hier zum erstenmal untersucht. Kriterien zur Bestimmung der Art der Verformung wurden auch angegeben.

ТЕПЛОВАЯ КОНТАКТНАЯ ПРОВОДИМОСТЬ. ТЕОРЕТИЧЕСКИЕ ИССЛЕДОВАНИЯ

Аннотация—Рассматривается тепловая контактная проводимость номинально плоских контактирующих поверхностей. Основное внимание в работе уделяется влиянию вида деформации на величину проводимости. Выведены выражения для тепловой проводимости для случаев: (1) чисто пластической деформации, (2) пластической деформации шероховатостей и упругой деформации слоя, лежащего ниже, и (3) чисто упругой деформации. Последние два важных случая рассматриваются впервые. Представлены критерии, определяющие вид деформации.